Mitigation of GPS Multipath Using Polarization and Spatial Diversities

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BIOGRAPHIES

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ABSTRACT

This paper presents a GPS multipath mitigation method using a dual circularly polarized antenna array and a multi-channel receiver. The method is based on the exploitation of both polarization and spatial diversity associated with a GPS signal and its multipath signals available at the receiver input. Conventional GPS antennas are right-hand circularly polarized (RHCP) to suppress multipath contributions to the input. This polarization-based discrimination of multipath signal cannot completely eliminate multipath induced GPS range measurement errors. We present an algorithm that performs spatial processing on the input from the left-hand circular polarized (LHCP) array with an increased relative strength of the multipath signal, thereby providing improved multipath angle of arrival (AOA) estimation. With the known multipath AOA and direct signal AOA (which can be obtained from almanac/ephemeris together with the antenna attitude or estimated in a separate process), we can then take advantage of the spatial diversity of the direct signal and multipath by applying null-steering to the RHCP array input. The paper presents the algorithm and simulation results for a uniform linear array receiving one direct signal and one multipath. Our preliminary studies showed that the multipath AOA estimator produces negligible error if the direct signal and multipath AOA are not close to each other (more than 5 degrees apart) and that the direct signal is not at low elevation. The results also suggested that longer time delay between the direct and multipath signal will increase multipath AOA estimation error but this increase is tolerable. Furthermore, we demonstrated that the multipath estimation improves with increasing spatial diversity for multipath and direct GPS signals even if the signal arrivals are close in time. Finally, we demonstrated that the multipath mitigation technique does produce an improved receiver correlator function which directly impacts the GPS code range measurement accuracy.

1. INTRODUCTION

Multipath is a major error source in GPS range measurements [1]. Many techniques have been developed to mitigate multipath error. These techniques include special consideration in antenna hardware design [2], receiver site selection [6][15], GPS tracking loop design [14][16][17], time-frequency domain signal processing of GPS code/carrier measurements [5][19], multi-channel receivers and/or spatial processing techniques [3][8][10][11][12], and the use of multipath polarization properties [18]. Many of these methods are only applicable to static applications which require the receiver remain in one position over extended time period, while others have difficulties in separating the multipath error from other error sources such as ionospheric delay errors. Spatial processing techniques alone are limited when the multipath and direct signal AOA are close. Multipath that is close to direct signal in time of arrival poses even bigger challenges. In this paper, we present a method that uses a dual circularly polarized antenna array and a multi-channel GPS receiver to mitigate multipath error by jointly exploiting both spatial and polarization diversity between a direct GPS and its multipath signals.

Direct GPS signals are right hand circular polarized (RHCP). When a GPS signal is reflected from a surface, a significant portion of it becomes left hand circular polarized (LHCP) when the angle of incidence with the surface from which it reflects exceeds the Brewster angle. Field studies of multipath in model urban environments show that detectable multipath signals do have significant incident angles [13], and thereby supports the generally accepted notion that multipath produced by a single reflection tend to be predominantly LHCP. This difference in the polarization states between the direct and multipath signals can be detected by a RHCP antenna, which will act to filter out most of the multipath signal, while allowing the direct signal to pass through. This improvement in the direct to multipath signal gain is the reason why GPS antennas are typically RHCP.

Using polarization alone to discriminate against multipath may not be sufficient to eliminate the multipath error in range measurements. Spatial diversity is another important means we can apply to further reduce multipath contribution. Spatial diversity occurs when the direct and multipath signal have different AOAs. The difference in the AOAs makes it possible to devise spatial processing techniques that will act to reduce the gain from the direction of the multipath signal while increasing the gain in the direction of the direct GPS signal. It is particularly useful for those cases where the path length difference between the direct and multipath signal is small. These are perhaps the most difficult cases in multipath mitigation, since the parametric estimation of the direct path delay in such cases becomes ill-conditioned. In such a case, spatial diversity may be the only effective means of mitigating multipath.

The main obstacle in employing spatial diversity is that the multipath AOA is not known and varies for a mobile platform. Multipath AOA must be estimated from the receiver input signal dynamically. The direct signal, which acts as an interference source in this case, decreases the effective signal to interference ratio for the multipath signal and therefore the accuracy of the multipath AOA estimation. One solution to this problem is to use the LHCP array to detect and estimate multipath. The LHCP antenna acts in the same way the RHCP antenna did, except this time instead of reducing the multipath relative to the direct signal, it reduces the strength of the direct signal relative to the multipath. Thus, the LHCP portion of the dual circularly polarized array weakens the direct signal, so we can obtain improved estimates of the multipath AOA.

The method presented in this paper uses the LHCP channel inputs of a dual polarization antenna array to estimate multipath AOA. The estimated multipath is then mitigated by applying spatial processing techniques to the input signal obtained from the RHCP channels. [7][18] have suggested multipath mitigation using both spatial processing and polarization discrimination, but no details (methodology, algorithms, or results) were given therein. The only a priori knowledge required by this method is information typically available from the ephemeris and the receiver tracking loop of the compounded direct and multipath signal. The ephemeris provides the approximate AOA of the direct signal, while the tracking loop generates approximate code phase and carrier reference signal. In this paper, we apply this method to a simple signal model that contains a direct signal, a multipath signal, and random channel noise. The methodology, however, is applicable to inputs that contain several multipath signals.

The remaining paper is organized as follows. Section 2 establishes the signal model, mathematical notations, and the assumptions. Section 3 gives a detailed methodology description. Section 4 presents simulation results and the performance evaluation of this method. Section 5 summarizes the method and the findings as well as future research in extending this method to more general scenarios.

2. SIGNAL MODEL, NOTATIONS, AND ASSUMPTIONS

We consider an M element dual circularly polarized linear array and a 2M-channel GPS receiver RF front end (see Fig.1). We assume the antenna elements are isotropic antennas to simplify the mathematics. Factors such as mutual coupling, realistic antenna element gain patterns and frequency responses, as well as possible antenna phase center shift and channel biases among others are not taken into consideration here. Most of these error terms can be calibrated out or estimated jointly, which we will address in separate papers.

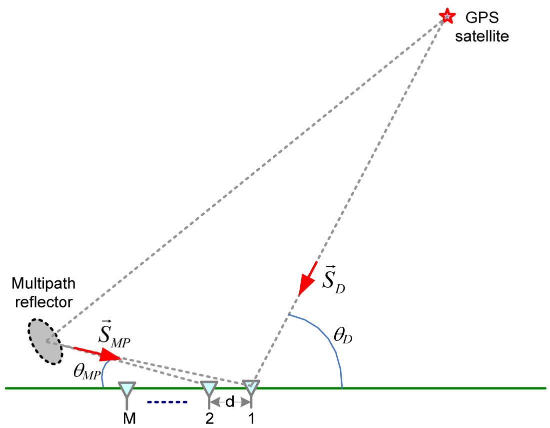


Fig. 1 Schematics of the analysis scenario

We assume an input signal contains one direct GPS signal, a multipath signal, and random white channel noise. We also assume that the GPS receiver RF front end outputs digitized baseband sample streams which can be converted to complex signals containing I and Q channel samples prior to spatial processing. The RHCP and LHCP channel signals at the k[th] sample (for k=1,…N) as:

$$\begin{aligned}\vec{\mathbf{X}}_{\mathbf{R}}(k) &= \upsilon_D(k)\vec{\mathbf{S}}_{\mathbf{D}} + \kappa_R\,\upsilon_{MP}(k)\vec{\mathbf{S}}_{\mathbf{MP}} + \vec{\boldsymbol{\varepsilon}}_{\mathbf{R}}(k)\\ \vec{\mathbf{X}}_{\mathbf{L}}(k) &= \kappa_L\,\upsilon_D(k)\vec{\mathbf{S}}_{\mathbf{D}} + \upsilon_{MP}(k)\vec{\mathbf{S}}_{\mathbf{MP}} + \vec{\boldsymbol{\varepsilon}}_{\mathbf{L}}(k)\end{aligned} \quad (1)$$

In equation (1), the subscripts "L" and "R" denote quantities associated with the RHCP and LHCP channel inputs, while the subscripts "MP" and "D" denote quantities associated with the multipath and direct signals, respectively. The $\vec{\mathbf{S}}$ vectors refer to the steering vectors of the sources; while the quantity υ represents the time dependent portion of the signal (i.e. carrier signal times the CA code). κ is a factor representing signal loss due to antenna polarization mismatch and reflection. Without loss of generality and for the sake of simplicity in analysis, we will use $\kappa_R = \kappa_L = 0.3$ throughout the paper. The random channel signals, given by the quantities ε, will be modeled as independent and identically distributed complex white Gaussian noise (CWGN) that have unity variance and are uncorrelated both spatially and temporally.

We assume that the GPS receiver is already tracking the compounded input signal and that the receiver has knowledge of ephemeris, and therefore, the approximate direct signal AOA. The receiver tracking loop also provides the prompt code and carrier reference signal which will be used in the algorithm described in the following section.

3. METHODOLOGY AND ALGORITHM

Our algorithm is based on the following 3-step algorithm:

1) Detect the multipath signal in the LHCP input by correlating the LHCP channel signal with the tracking loop reference signal.
2) Estimate the multipath AOA from Step 1.
3) Use the estimated multipath AOA and known direct signal AOA to perform null-steering on the RHCP array signal.

In the remainder of this section, we discuss each of these steps in detail.

1) Detecting the Multipath Signal in the LHCP Input.

The LHCP signal has more gain for the multipath component than for the direct signal. Our goal is to manipulate them to estimate the multipath AOA. Since both the direct GPS signal and its multipath are far below the noise floor, signal detection will be performed first. This can be achieved by correlating the array signal, channel wise, with the known tracking loop prompt code and carrier reference. We denote the compounded signal tracking loop reference sequence as $\hat{\text{r}}(\text{k})$.

The correlated output, which we will denote by $\vec{\mathbf{Y}}_{\text{L}}$, can be easily expressed as:

$$\vec{\mathbf{Y}}_{\mathbf{L}} = \frac{\sum_{k=1}^{N}\vec{\mathbf{X}}_{\mathbf{L}}(k)\hat{r}^*(k)}{N} \quad (2)$$

where * represents the complex conjugate operation, and N is number of samples used in the correlation operation.

Substituting the LHCP channel signal model in (1) into (2), we can rewrite $\vec{\mathbf{Y}}_{\text{L}}$ as:

$$\vec{\mathbf{Y}}_{\mathbf{L}} = \kappa_{\text{L}}\,\rho_{\text{D}}\,\vec{\mathbf{S}}_{\mathbf{D}} + \rho_{\text{MP}}\,\vec{\mathbf{S}}_{\mathbf{MP}} + \vec{\boldsymbol{\eta}}_{\mathbf{L}} \quad (3)$$

where

$$\rho_{MP} = \frac{\sum_{k=1}^{N}\upsilon_{MP}(k)\hat{r}^*(k)}{N}$$

$$\rho_{D} = \frac{\sum_{k=1}^{N}\upsilon_{D}(k)\hat{r}^*(k)}{N}$$

$$\eta_{\text{L}}(m) = \frac{\sum_{k=1}^{N}\varepsilon_{\text{L}}(m,k)\hat{r}^*(k)}{N} \qquad (m = 1,\ldots, M)$$

The increased signal strength in $\vec{\mathbf{Y}}_{\text{L}}$ can now be used to estimate multipath AOA.

We would like to point out here that using unit modulus reference signal implies that $\vec{\boldsymbol{\eta}}_{\mathbf{L}}$ is a complex normal random vector with zero mean and covariance $\Sigma = \frac{1}{N} I_M$ (where I_M = M×M identity matrix).

2) Multipath AOA Estimation.

The main quantity used in estimating the multipath AOA is a beamforming weight which has the following form:

$$\hat{\mathbf{w}}_\mathbf{L} = \frac{P_D^\perp \hat{\mathbf{S}}_\mathbf{MP}}{\sqrt{\hat{\mathbf{S}}_\mathbf{MP}^H P_D^\perp \hat{\mathbf{S}}_\mathbf{MP}}} \qquad (4)$$

where the superscript *H* denotes the complex conjugate of a vector and hats on all vector quantities denote estimates, not unit vectors. $P_D^\perp$ is the projection matrix onto the subspace orthogonal to $\vec{\mathbf{S}}_\mathbf{D}$ and $\hat{\mathbf{S}}_\mathbf{MP}$ is an estimate for the multipath steering vector.

As long as $\hat{\mathbf{S}}_\mathbf{MP} \neq \vec{\mathbf{S}}_\mathbf{D}$, $\hat{\mathbf{w}}_\mathbf{L}$ is well-defined. The method for estimating the multipath AOA is based on the following proposition:

$$\hat{\mathbf{w}}_\mathbf{L} = \underset{\vec{\mathbf{w}} \in X^M / \{\vec{\mathbf{S}}_\mathbf{D}\}}{ArgMax} \left\{ E\left[\left| \vec{\mathbf{w}}^H \vec{\mathbf{Y}}_\mathbf{L} \right|^2 \right] \right\}$$

$$iff$$

$$\hat{\mathbf{S}}_\mathbf{MP} = \vec{\mathbf{S}}_\mathbf{MP} \text{ and } \vec{\mathbf{S}}_\mathbf{MP} \neq \vec{\mathbf{S}}_\mathbf{D}$$

We present the proof below:

Proof:

The backwards implication is a well-known result that can be found elsewhere in the literature [4], so we prove just the forwards implication. Let us assume the form of $\vec{Y}_L$ given in equation (3) and the hypothesis of this proposition is true. Since by construction, $\hat{\mathbf{w}}_\mathbf{L}$ is orthogonal to $\vec{\mathbf{S}}_\mathbf{D}$ for all allowed multipath steering vector estimates, it follows that:

$$\hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{Y}}_\mathbf{L} = \rho_{MP} \hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{S}}_\mathbf{MP} + \hat{\mathbf{w}}_\mathbf{L}^H \vec{\boldsymbol{\eta}}_\mathbf{L} \qquad (5)$$

Furthermore, because the matrix $P_D^\perp$ is both idempotent and Hermitian, it is easy to verify that $\hat{\mathbf{w}}_\mathbf{L}$ is a unit vector. This allows us to conclude that the noise term reduces to a single scalar, e_L, which has the distribution CWGN(0,1/N). This gives us the final expression:

$$\hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{Y}}_\mathbf{L} = \rho_{MP} \hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{S}}_\mathbf{MP} + e_L \qquad (6)$$

Taking the expected value of the squared norm, we find:

$$E\left[\left| \hat{\mathbf{w}}^H \vec{\mathbf{Y}}_\mathbf{L} \right|^2 \right] = \left| \rho_{MP} \right|^2 \left| \hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{S}}_\mathbf{MP} \right|^2 + \frac{1}{N} \qquad (7)$$

From which we see that $E\left[\left| \hat{\mathbf{w}}^H \vec{\mathbf{Y}}_\mathbf{L} \right|^2 \right]$ is maximized iff $\left| \hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{S}}_\mathbf{MP} \right|^2$ is maximized.

If we consider the quantity $\left| \hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{S}}_\mathbf{MP} \right|^2$, we note that it can be rewritten as:

$$\left| \hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{S}}_\mathbf{MP} \right|^2 = \left\{ \frac{\left| \hat{\mathbf{S}}_\mathbf{MP}^H P_D^\perp \vec{\mathbf{S}}_\mathbf{MP} \right|^2}{\left| P_D^\perp \hat{\mathbf{S}}_\mathbf{MP} \right| \left| P_D^\perp \vec{\mathbf{S}}_\mathbf{MP} \right|} \right\} \left| P_D^\perp \vec{\mathbf{S}}_\mathbf{MP} \right|^2 \qquad (8)$$

Since the factor $\left| P_D^\perp \vec{\mathbf{S}}_\mathbf{MP} \right|^2$ is a fixed quantity, we see that $\left| \hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{S}}_\mathbf{MP} \right|^2$ is maximized when the bracketed quantity on the RHS of equation (8) is maximized. It is known [9] that under the conditions stated in this proof that the term inside the bracket is an inner product, and hence, it satisfies the Cauchy-Schwarz inequality, which in our case, can be written in the form:

$$\frac{\left| \hat{\mathbf{S}}_\mathbf{MP}^H P_D^\perp \vec{\mathbf{S}}_\mathbf{MP} \right|^2}{\left| P_D^\perp \hat{\mathbf{S}}_\mathbf{MP} \right| \left| P_D^\perp \vec{\mathbf{S}}_\mathbf{MP} \right|} \leq 1 \qquad (9)$$

This quantity is maximized when equality holds in the above relationship. But from the Cauchy-Schwarz inequality, it is known that equality holds iff the estimated and true multipath steering vectors are related by a scalar. Since the steering vectors are parametrically defined as phase factors, this requires the two steering vectors to be related by a constant phase factor. QED

The significance of this result is that we now have a simple way to estimate the multipath AOA. If we perform a grid search over the one dimensional space of possible multipath AOA values, construct the steering vector for each AOA, and then using the steering vector to construct the weight $\hat{\mathbf{w}}_\mathbf{L}$ according to (4), we can compute the value of the quantity $\left| \hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{Y}}_\mathbf{L} \right|^2$ for each multipath AOA. The previous theorem tells us that the choice of the AOA with the largest value of $\left| \hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{Y}}_\mathbf{L} \right|^2$ will yield the true multipath AOA.

Figure 2 shows a simulation result from a grid search for a five element ULA. The solid curve shows the result of the grid search when the input signal contains no noise. The maximum value of $\left| \hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{Y}}_\mathbf{L} \right|^2$ occurs exactly at the true multipath AOA (within the grid resolution of 0.1°). The black curve is generated using the same signal except now random channel noise is added. In this case, the (pre-correlation) multipath SNR is -28 dB, and we see that the location of the maximum peak has an offset of about 2.6° from the true AOA. Although the distribution of the AOA values remains under investigation, we can prove that the estimator for the maximum peak height is asymptotically unbiased and consistent.

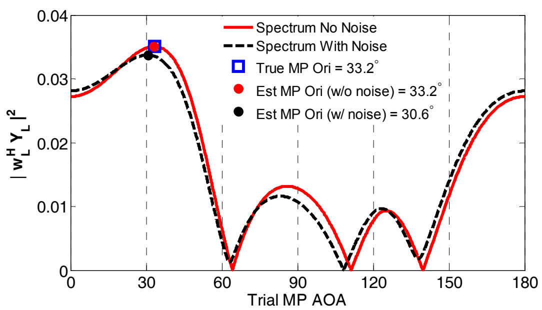


Fig. 2. Simulation results demonstrating the effectiveness of the multipath AOA estimator.

It is important to point out here that the grid search involved in this method is not computationally intensive for two reasons. First, the search is performed on post-correlation vectors whose size is limited by the array dimension. Second, the multipath AOA estimation is intended for the purpose of null-steering. As shown by the analysis in [4], the power leakage due to an error in the multipath AOA estimation is on the same order as the product of the power ratio of multipath to direct signal and the square of the AOA error (in radians). This can be a serious problem if we are dealing with high power interferences. However, since the multipath signal is typically much weaker than the direct signal, a coarse grid search of 1° gives an AOA estimation error that will not significantly affect our final null-steering result. It can be shown for a linear array of M elements, a grid search with a resolution of Δ° requires 720×M/Δ° multiplications for the entire search operation, which is a reasonable number in terms of computational cost.

3) Null-Steering on the RHCP Channel Signal Using the Multipath AOA

Using the multipath AOA estimation obtained with the LHCP array input, we can form the estimated multipath steering vector. Using this estimated multipath steering vector and known direct signal steering vector, we can now construct the optimal null-steering weight for the RHCP array input:

$$\hat{\mathbf{w}}_\mathbf{R} = P_{MP}^{\perp}\vec{\mathbf{S}}_\mathbf{D} \tag{10}$$

where $P_{MP}^{\perp}$ is the projection matrix onto the subspace orthogonal to $\vec{\mathbf{S}}_\mathbf{MP}$. The final output signal that has the reduced multipath contribution is:

$$\begin{aligned} y_R(k) &= \hat{\mathbf{w}}_\mathbf{R}^H \vec{\mathbf{X}}_\mathbf{R}(k) \\ y_R(k) &= \upsilon_D(k)\left(\vec{\mathbf{S}}_\mathbf{D}^H P_{MP}^{\perp}\vec{\mathbf{S}}_\mathbf{D}\right) + \vec{\mathbf{S}}_\mathbf{D}^H P_{MP}^{\perp}\vec{\boldsymbol{\eta}}_\mathbf{R}(k) \end{aligned} \tag{11}$$

We present our preliminary simulation results on the above algorithm performance in the next section.

4. SIMULATION RESULTS

The main factor affecting the performance of the method presented in Section 3 is the accuracy of the multipath AOA estimation. Therefore, our principal aim in this section will be to use simulations to evaluate the performance of the algorithm to estimate the multipath AOAs.

We can better understand how the various signal parameters will affect the results from our estimations by analyzing the quantity $E\left[\left|\hat{\mathbf{w}}^H \vec{\mathbf{Y}}_\mathbf{L}\right|^2\right]$ used in the grid search to estimate the multipath AOA. From equation (7), we notice that for a fixed number of data points, $E\left[\left|\hat{\mathbf{w}}^H \vec{\mathbf{Y}}_\mathbf{L}\right|^2\right]$ is effectively the product of two factors. The first factor, $|\rho_{MP}|^2$ is a direct measure of the multipath post-correlated signal's strength (relative to the noise). If we use the fact that the correlation for multipath signals close in time to the direct signal is a tent-function, then we can write this factor as:

$$|\rho_{MP}|^2 = P_{MP}\left(1 - \frac{\delta n}{\Delta n}\right)^2 \tag{12}$$

where P_{MP} is the multipath signal's (input) power, δn is the CA code offset between the multipath and reference signal, and Δn is the number of data points in 1.5 chip lengths (which in our system, turns out to be 7). The second factor, $\left|\hat{\mathbf{w}}_\mathbf{L}^H \vec{\mathbf{S}}_\mathbf{MP}\right|^2$, is a direct measure of how close the multipath and direct source steering vectors are. To derive the relationship that shows this, we note that similar to the inner product defined in the previous proof, we can also define the following direction cosine:

$$\cos\gamma = \frac{\left|\vec{\mathbf{S}}_\mathbf{MP}^H \vec{\mathbf{S}}_\mathbf{D}\right|}{\sqrt{\left(\vec{\mathbf{S}}_\mathbf{MP}^H \vec{\mathbf{S}}_\mathbf{MP}\right)\left(\vec{\mathbf{S}}_\mathbf{D}^H \vec{\mathbf{S}}_\mathbf{D}\right)}} = \frac{\left|\vec{\mathbf{S}}_\mathbf{MP}^H \vec{\mathbf{S}}_\mathbf{D}\right|}{M} \tag{13}$$

If we now consider the quantity of interest, we find it can be expressed in terms of this direction cosine as follows:

$$\left|\hat{\mathbf{w}}_{\mathbf{L}}^{\mathrm{H}}\vec{\mathbf{S}}_{\mathbf{MP}}\right|^2 = \left(\frac{\vec{\mathbf{S}}_{\mathbf{MP}}^H \mathrm{P}_{\mathrm{D}}^{\perp}\vec{\mathbf{S}}_{\mathbf{MP}}}{\sqrt{\vec{\mathbf{S}}_{\mathbf{MP}}^H \mathrm{P}_{\mathrm{D}}^{\perp}\vec{\mathbf{S}}_{\mathbf{MP}}}}\right)^2 = \vec{\mathbf{S}}_{\mathbf{MP}}^H \mathrm{P}_{\mathrm{D}}^{\perp}\vec{\mathbf{S}}_{\mathbf{MP}} \quad (14)$$

The projection matrix can be expressed as:

$$\mathrm{P}_{\mathrm{D}}^{\perp} = I_M - \frac{1}{M}\vec{\mathbf{S}}_{\mathbf{D}}\vec{\mathbf{S}}_{\mathbf{D}}^H \quad (15)$$

Substituting for $\mathrm{P}_{\mathrm{D}}^{\perp}$ into equation (14)

$$\left|\hat{\mathbf{w}}_{\mathbf{L}}^{\mathrm{H}}\vec{\mathbf{S}}_{\mathbf{MP}}\right|^2 = M - \frac{1}{M}\left|\vec{\mathbf{S}}_{\mathbf{D}}^H\mathbf{S}_{\mathbf{MP}}\right|^2 \quad (16.a)$$

and use of equation (13) gives us:

$$\left|\hat{\mathbf{w}}_{\mathbf{L}}^{\mathrm{H}}\vec{\mathbf{S}}_{\mathbf{MP}}\right|^2 = M - M\cos^2\gamma \quad (16.b)$$

Defining sinγ as $\sqrt{1-\cos^2\gamma}$, it thus follows that:

$$\left|\hat{\mathbf{w}}_{\mathbf{L}}^{\mathrm{H}}\vec{\mathbf{S}}_{\mathbf{MP}}\right|^2 = M\sin^2\gamma \quad (16.c)$$

Combining both factors now, we see that the principal quantity of interest can be expressed totally in terms of the signal parameters on which it depends as:

$$E\left[\left|\hat{\mathbf{w}}^H\vec{\mathbf{Y}}_{\mathbf{L}}\right|^2\right] = \mathrm{M}\,\mathrm{P}_{\mathrm{MP}}\left(1-\frac{\delta n}{\Delta n}\right)^2\sin^2\gamma \quad (17)$$

This result gives a simple expression that shows how the polarization diversity (which here is given by the net multipath power obtainable from the array, M P_{MP}), the time diversity (given by the square of the tent function), and the spatial diversity (given by the $\sin^2\gamma$ term) affect the quantity from which the multipath AOA is estimated.

We conducted a series of simulations to analyze the impact of these factors on the multipath AOA estimator. These simulations were carried out with a 5 element ULA. The SNR of the direct and multipath signals are -18dB and -23dB, respectively. The receiver RF front end sampling frequency is 5 MHz and 20 msec data (1 GPS data bit) or 100K samples are used in analysis. A variable in the simulations is the CA code offset between the direct and multipath signal. Given the sampling frequency, there are 7 samples in 1.5 CA code chip lengths (which is typically the range in the time delay where the effects of multipath are most significant). We allow the CA code offset to vary from 0 to 3 samples. For ease of analysis, we will only investigate the scenario in which the path length difference between the direct signal and multipath signal is independent of their AOAs. With this consideration, we can compute the phase difference between the two signals using the path length difference. All simulations are performed using 250 Monte-Carlo runs. Because we shall refer to the angles of arrival of the direct and multipath signals throughout this analysis, we shall introduce the notation for these quantities as θ_D and θ_{MP}, respectively.

As equation (17) predicts, we will see that the two major factors affecting the quality of the multipath AOA estimates will be the time delay between the reference and multipath signal and the relative orientations of the direct and multipath steering vectors. In our first simulation, we let the time delay between the direct signal and multipath to be 0. We choose 3 different direct signal AOAs at 10°, 40°, and 70° relative to the array axis, respectively. We computed the accuracy of their multipath AOA estimation for all θ_{MP} in the 0 to 90^o range (in 2° increments). Fig. 3 plots the multipath AOA estimation error as a function of θ_{MP}. One immediately can see that the multipath AOA estimation error is largest when θ_{MP} and θ_D are close to each other. Also notice that when θ_D is 40° and 70°, even the largest multipath AOA estimation has only small deviations on the order of the grid search resolution (of 1°) from the true AOA. In the case where θ_D =10°, however, the deviation is pronounced when the direct signal and multipath AOA are close. This demonstrates that multipath AOA estimation is more error prone for low elevation incidence.

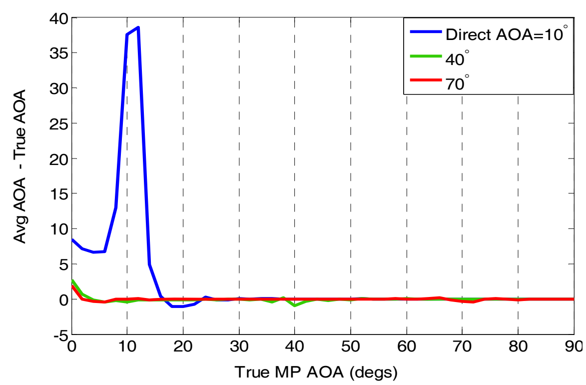


Fig.3 Simulate multipath AOA estimation error as a function of true multipath AOA for three different direct signal AOAs.

A second observation we see from Figure 3 is that the multipath AOA estimation error increases as θ_{MP} approaches zero. This result is an artifact of the grid search method. As θ_{MP} approaches zero, the grid search function about θ_{MP} "flattens out" as shown in figure 4. To validate this argument, we plot the grid functions for a multipath source with a fixed AOA of 5^o, for three different θ_D values of 20^o, 50^o, and 80^o, respectively. Figure 4 shows that for all three θ_D values, the multipath AOA estimator is flat in the vicinity of the true multipath AOA (at 5^o). Notice that the flatness is indeed affected by θ_D: as it increase, the extent over which the peak is flat decreases.

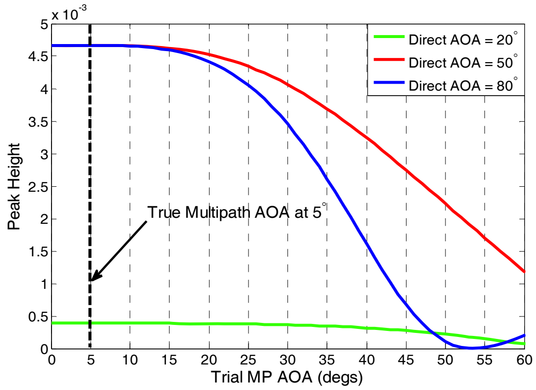


Fig.4 Multipath AOA estimator as a function of true multipath AOA for three different θ_D values.

The standard deviation of the multipath AOA error follows a similar trend. The top panel of Figure 5 shows that except for the case when the direct and multipath AOAs are both very small, the standard deviation is acceptably low for all multipath AOA estimation if $|\theta_D - \theta_{MP}| > 5°$. The lower panel of Figure 5 is generated to illustrate the origin of the standard deviation. For each of the three direct signal AOAs, we plot $\sin^2\gamma$ as a function of the θ_{MP}, where γ is angle between the direct and multipath steering vectors. As we showed previously, the term $\sin^2\gamma$ captures all of the multipath and direct signal AOA dependence in the multipath AOA estimator. When the multipath AOA is close to that of the direct source, $\sin^2\gamma$ monotonically goes to zero as $\theta_{MP} \rightarrow \theta_D$.

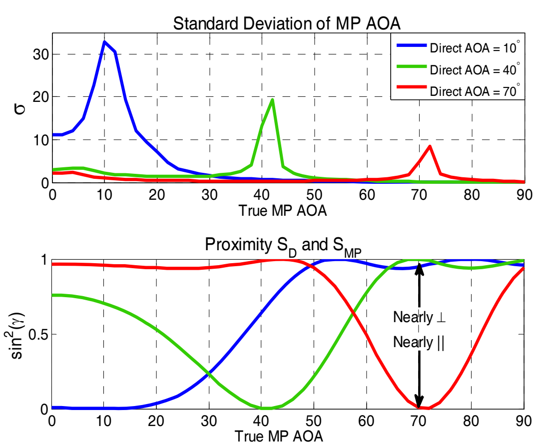


Fig. 5 Multipath AOA estimation standard deviation as a function of true multipath AOA for three different θ_D values.

As a consequence, when null-steering is applied to the direct source, an increasingly larger portion of the multipath is also removed. This effectively reduces the multipath signal strength, which, in turn, leads to a larger standard deviation in the multipath AOA estimation.

The above simulation results imply that when two signals that are replicas of each other and when they occur close in both time and space, it is difficult to selectively remove just one of them or estimate a specific signal's parameters. This should not come as a surprise.

If a multipath and its direct counterpart have a larger time delay, the multipath AOA estimation will have larger error. This is because mismatch between the reference and the multipath signal will be more sever, leading to a loss in the effective post-correlation multipath signal strength. Figure 6 compares the error and standard deviation for two scenarios where the multipath and direct signal time delay is half a code chip and 0 respectively. The results indicate that as long as $|\theta_D - \theta_{MP}| > 5°$, the multipath AOA estimation is acceptable.

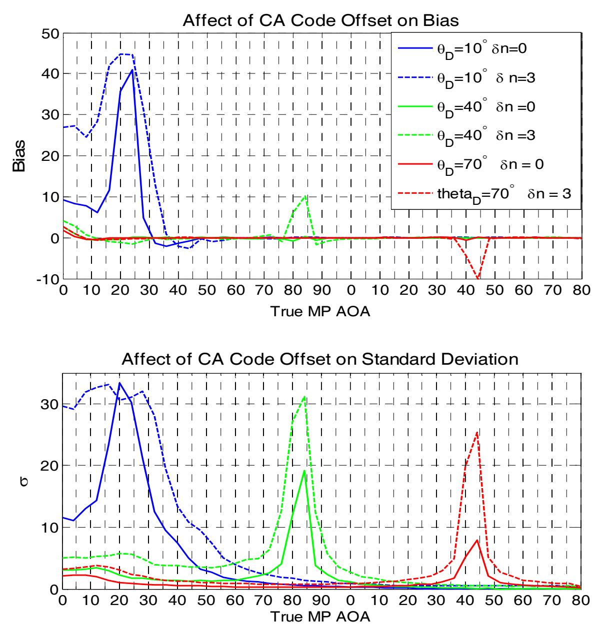
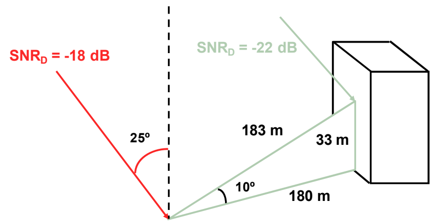


Fig. 6 Multipath AOA estimation error and standard deviation dependency on the delay time between direct and multipath signal.

A final simulation is performed to validate the effectiveness of the multipath mitigation algorithm by examining the direct signal correlator output which directly impacts the range measurement accuracy. Figure 7 is a schematic of the direct signal and multipath used in the simulation. The direct signal elevation is 75^o while the multipath AOA is 10^o. The time delay between the two signals is 2.4

sample intervals. Figure 8 shows two correlator outputs. The black curve is generated using the original input while the red curve is produced using the output after applying the multipath mitigation algorithm presented in this paper. The effect of multipath mitigation on the RHCP antenna input is evident.



$$\left.\begin{matrix}(\phi_D,\theta_D)=(30,25)\\(\phi_M,\theta_M)=(113,80)\end{matrix}\right\}\Rightarrow \gamma=94^\circ \quad \text{delay}=145\text{ m} \;\Rightarrow\; \delta n = 2.4$$

Fig. 7 Schematic of a simulation scenario

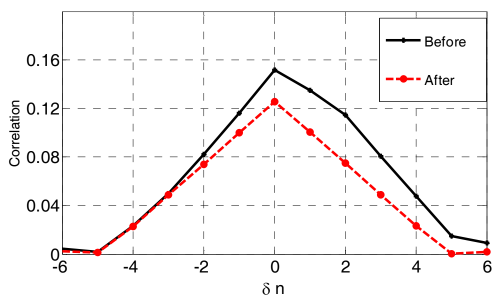


Fig. 8 Comparison of correlator output before and after multipath removal

5. CONCLUSIONS AND FUTURE WORKS

This paper presented a GPS multipath mitigation method that utilizes a dual circular polarized antenna array and a multichannel GPS receiver. The LHCP array input was used to detect and estimate the multipath signal AOA. Using the estimated multipath AOA and known direct signal AOA, null-steering was applied to the RHCP array input to minimize multipath contribution. This method therefore takes full advantage of both spatial and polarization diversity of the direct signal and multipath signal. The paper provided mathematical proofs for the method and detailed algorithms.

Simulations were performed to evaluate the effectiveness of the method for the simple scenario involving one direct signal, one multipath, and random channel noises. A uniform linear array with ideal and isotropic antenna elements were assumed in the analysis and simulation. The results showed that the multipath AOA estimator produces negligible error and standard deviation if $|\theta_D - \theta_{MP}| > 5°$ and that the direct signal AOA is not near 0^o. The results also showed that longer time delay between the direct and multipath signal increases multipath AOA estimation error but this increase is tolerable. Finally, we demonstrated that the multipath mitigation technique does produce improved receiver correlator function which directly impacts the GPS code range measurement accuracy.

There are a number of issues that are worthy of continued studies. First, although the simulation is performed for an ULA with idealized antenna element and a simple signal model containing only one direct signal and multipath, the basic idea should apply to 2D array and multiple multipath signals in input. Second, we have derived an optimized beam forming weight that minimizes multipath contribution to the RHCP input and provides an alternative means to the null-steering approach used in this paper. The advantage of this optimization approach is that it eliminates the need for the grid search operation we presented in this paper. We plan to carry out more in-depth study on the performance of the optimization method. Third, the impact of mutual coupling and other mismatching factors on the performance of the method needs to be investigated.

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REFERENCES:

[1] Braasch, M., “Multipath effects,” in *Global Positioning System: Theory and Applications,* edited by Parkingson B. W. and Spilker J. J. Jr., AIAA Publisher, vol. 1, Chp.14, pp. 547-568, 1996.

[2] Counselman, C. C., “Array antennas for DGPS,” *Proc. IEEE PLANS*, pp. 352-357, Palm Springs, CO, Apr. 1998.

[3] Fante R. L. and J. J. Vaccaro, “Evaluation and reduction of multipath-induced bias on GPS time-of-arrival,” *IEEE Trans. Aero. Elec. Sys., vol.*39, pp. 911-920, 2003.

[4] Friedlander B. and B. Porat. “Performance analysis of a null-steering algorithm,” *IEEE Trans. Acoustics, Speech and Signal Processing*, Vol. 37, No. 4, pp. 461-466, 1989.

[5] Greenspan, R., A. Ng, J. Przyjemski, and J. Veale, “Accuracy of relative positioning by interferometry with reconstructed carrier GPS: experimental results,” *Proc. 3rd Int. Geodetic Sym. Satellite Doppler Positioning*, pp. 1177-1195, Las Cruces, NM, Feb. 1982.

[6] Lachapelle, G., W. Falkenberg, D. Neufeldt, and P. Keilland, "Marine DGPS using code and carrier in a multipath environment," *Proc. ION-GPS*, pp. 343-347, Colorado Spring, CO., Sept. 1989.
[7] Manandhar, D., R. Shibasaki, P. Normark, "GPS signal analysis using LHCP/RHCP antenna and software GPS receiver," *Proc. ION-GNSS*, Long Beach, CA, Sept. 2004.
[8] Moelker, D., "Multipath antennas for advanced GNSS multipath mitigation and multipath direction finding," *Proc. ION-GPS*, pp. 541-550, Kansas City, KS, Sept. 1997.
[9] Ortgea, J.M., *Matrix Theory: a Second Course The University Series in Mathematics*, Plenum Press, NY and London, 1987.
[10] Raquet, J. and G. Lachapelle, "Determination and reduction of GPS reference station multipath using multiple receivers," *Proc. ION-GPS*, pp. 673-681, Kansas City, KS, Sept. 1996.
[11] Ray, J. K., "Use of multiple antennas to mitigate carrier phse multipath in reference stations," *Proc. ION-GPS*, pp. 269-279, Nashville, TN, Sept. 1999.
[12] Seco, G. and J. Fernadez-Rubio, "Array signal processing techniques for pseudorange and carrier phase measurement," *Proc. 2nd European Symp. GNSS*, pp. 20-23, vol.IX-P-10, 1998.
[13] Steingass, A. and Lehner, A., "Navigation in multipath environments for suburban applications", *Proc. ION GNSS*, Ft. Worth, TX, Sept. 2007.
[14] Townsent, B. and P. Fenton, "A practical approach to the reduction of pseudorange multipath errors in a L1 GPS receiver," *Proc. ION-GPS*, pp. 143-148, Salt Lake City, UT, Sept. 1994.
[15] Tranquilla, J. and J. Carr, "GPS multipath field observations at land and water sites," *Navigation,* vol.37, pp. 393-414, 1991.
[16] Van Dierendonck, A. J., P. Fenton, and T. Ford, "Theory and performance of narrow correlator technology in GPS receiver," *Navigation*, vol.39, pp. 265-283, 1992.
[17] Van Nee, R., *Multipath and Multi-transmitter Interference in Spread-Spectrum Communication and Navigation Systems*, Delft University Press, Delft, the Netherlands, 1995.
[18] Yang, C. and A. Porter, "GPS multipath estimation and mitigation via polarization sensing diversity : parallel iterative cross cancellation", *Proc. ION-GNSS*, Long Beach, CA, pp. 2707-2719, 2005, Sept. 2005,
[19] Zhang, Y. and C. Bartone, "Multipath mitigation in the frequency domain," *Proc. IEEE PLANS* , pp. 486-495, Monterey, CA, April 2004.